

Modified Entropic Force

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The theory of statistical thermodynamics tells us the equipartition law of energy does not hold in the limit of very low temperatures. It is found the Debye model is very successful in explaining the experimental results for most of the solid objects. Motivated by this fact, we modify the entropic force formula which is proposed very recently. Since the Unruh temperature is proportional to the strength of gravitational field, so the modified entropic force formula is an extension of the Newtonian gravity to weak field. On the contrary, General Relativity extends Newtonian gravity to strong field case. Corresponding to Debye temperature, there exists a Debye acceleration g_D . It is found the Debye acceleration is $g_D = 10^{-15} \text{N} \cdot \text{kg}^{-1}$. This acceleration is very much smaller than the gravitational acceleration $10^{-4} \text{N} \cdot \text{kg}^{-1}$ which is felt by the Neptune and the gravitational acceleration $10^{-10} \text{N} \cdot \text{kg}^{-1}$ felt by the Sun. Therefore, the modified entropic force can be very well approximated by the Newtonian gravity in the solar system and in the Galaxy. With this Debye acceleration, we find the current cosmic speeding up can be explained without invoking any kind of dark energy.

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I. INTRODUCTION

Recently, Verlinde [1] makes one interesting proposal that gravity may be explained as an entropic force caused by the changes in the information associated with the positions of material bodies. The proposal is very interesting in the following two aspects. In the first place, with the assumption of the entropic force together with the Unruh temperature [2], Verlinde is able to derive the second law of Newtonian mechanics. Secondly, the assumption of the entropic force together with the holographic principle [3] and the equipartition law of energy can lead to the Newtonian law of gravitation. Similar discoveries are also made by Padmanabhan [4]. Padmanabhan finds that using the equipartition law of energy for the horizon and the thermodynamic relation $S = E/2T$, one can obtain the Newtonian law of gravity. Here S and T are thermodynamic entropy and the temperature associated with the horizon. E is the active gravitational mass which produces the gravitational field in the spacetime [5]. Actually, Jacobson has derived the Einstein's equations by employing the Clausius relation and the equivalence principle in 1995 [6].

Using the entropic force, Shu and Gong [7], Cai, Cao and Ohta [8] derived the Friedmann equation nearly simultaneously while with different methods. On the other hand, using this entropic force, Smolin [9] derived the Newtonian gravity in loop quantum gravity. Mäkelä [10] pointed out that Verlinde's entropic force is actually the consequence of a specific microscopic model of spacetime [11]. Similar ideas were applied to the construction of

holographic actions from black hole entropy by Caravelli and Modesto [12]. Li and Wang [13] showed that the holographic dark energy [14] can be derived from the entropic force formula.¹

We note that statistical thermodynamics reveals the equipartition law of energy does not hold in the very low temperatures. It is found the Debye model is very successful in explaining the experimental results for most of the solid objects. In particular, many experiments on Debye model indicate that the lower the temperature, the better the consistency is.

On the other hand, the quantum theory of black holes finds black holes satisfy the first law of thermodynamics [15]. Furthermore, the Unruh effect indicates that a gravitational field corresponds a thermal bath with temperature of T . So every gravitational system may be physically corresponding to a statistical thermodynamics system. Since the equipartition law of energy does not hold in low temperatures for thermodynamic system, we anticipate the entropic force formula should be modified for the very weak gravitational fields.

Actually, the strength of gravitational fields are $10^1 \text{N} \cdot \text{kg}^{-1}$ on the earth, $10^{-4} \text{N} \cdot \text{kg}^{-1}$ for the solar system (felt by the Neptune) and $10^{-10} \text{N} \cdot \text{kg}^{-1}$ for the Galaxy (felt by the Sun). In one word, the strength of gravitational field becomes smaller and smaller with the increasing of scales. Therefore, it may be plausible that the entropic force (or Newtonian gravity) takes a different form in the background of an extreme weak gravitational field.

In this paper, motivated by the Debye's model in ther-

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¹ Same observation was disclosed by Johannes Koelman in his blog. (see <http://www.scientificblogging.com-hammock> physicist-it bit how get rid dark energy)

modynamics, we extend the entropic force formula. With this modification, we find the resulting entropic force can speed up the expansion of the Universe. We shall use the system of units with $G = c = \hbar = k = 1$ and the metric signature $(-, +, +, +)$ throughout the paper.

II. MODIFIED ENTROPIC FORCE

Following [1], we consider a compact spatial region V with a compact boundary B which is a sphere. The compact boundary B plays the role of a holographic screen. The number N of bits for freedom on the screen is taken to be equal to the area A of the screen:

$$N = A. \quad (1)$$

Let g denotes the proper gravitational acceleration on the screen which is produced by the matter sources inside the screen B . Then the Unruh effect tells us the gravitational field would correspond to a system filled with thermal gas with the temperature of T for a proper observer:

$$T = \frac{g}{2\pi}. \quad (2)$$

So according to the equipartition law of energy, the total energy inside the screen is

$$E = \frac{1}{2}NT. \quad (3)$$

Verlinde assumes the energy is equal to the gravitational mass M inside the screen:

$$E = M. \quad (4)$$

Combining above equations, we can obtain the spherically symmetric and static gravitational field equations:

$$M = \frac{1}{2} \cdot 4\pi r^2 \cdot \frac{g}{2\pi}, \quad (5)$$

namely,

$$g = \frac{M}{r^2}. \quad (6)$$

It is exactly the Newtonian law of gravity. Statistical thermodynamics tells us the equipartition law of energy fails in the very low temperatures. It is found the Debye model is very successful for most of solid objects. Since a gravitational system may be physically corresponding to a statistical thermodynamics system and the strength of gravitational field becomes smaller and smaller with the increasing of cosmic scales, we anticipate the Newtonian gravity may be modified on the very large scale of the Universe.

Motivated by this point, we modify the equipartition law of energy as follows:

$$E = \frac{1}{2}NT\mathfrak{D}(x), \quad (7)$$

where the Debye function is defined by

$$\mathfrak{D}(x) \equiv \frac{3}{x^3} \int_0^x \frac{y^3}{e^y - 1} dy. \quad (8)$$

x is related to the temperature T as follows:

$$x \equiv \frac{T_D}{T}. \quad (9)$$

T_D is the Debye critical temperature. For a gravitational system, T_D is related to the critical gravitational acceleration. So we obtain the modified entropic force from Eq. (7) as follows

$$g = \frac{M}{r^2} \cdot \frac{1}{\mathfrak{D}(x)}, \quad (10)$$

with

$$x \equiv \frac{g_D}{g}. \quad (11)$$

The Debye acceleration is related to the Debye temperature

$$g_D = 2\pi T_D. \quad (12)$$

We note that when $x \ll 1$, i.e. in the limit of strong gravitational field, we have

$$\mathfrak{D}(x) = 1. \quad (13)$$

So the Newtonian gravity is recovered. On the other hand, in the limit of weak gravitational field, i.e. $x \gg 1$, we have

$$\mathfrak{D}(x) = \frac{\pi^4}{5x^3}, \quad (14)$$

and

$$g = \left(\frac{5Mg_D^3}{\pi^4} \right)^{\frac{1}{4}} \cdot \frac{1}{\sqrt{r}}. \quad (15)$$

In this limit, the gravitational field is very different from Newtonian gravity. In the next, let's look for the value of g_D from the considerations of cosmological physics. Using the method of Shu and Gong [7], we obtain the following equation

$$4\pi(\rho + p) = - \left(\dot{H} - \frac{K}{a^2} \right) \left[-2\mathfrak{D}(x) + \frac{3x}{e^x - 1} \right]. \quad (16)$$

Here ρ , p are the total energy density and total pressure of cosmic fluids, respectively. a , H are the scale factor and Hubble parameter, respectively. $K = 0, +1, -1$ describe the three kind of geometry of the Universe. Dot denotes the derivative with respect to the comoving time t . Taking into account the Hawking temperature T for the Universe [16, 17]

$$T = \frac{H}{2\pi}, \quad (17)$$

we find x can be rewritten as

$$x \equiv \frac{H_D}{H} . \quad (18)$$

Thus the critical Hubble parameter H_D is exactly the Debye acceleration g_D :

$$H_D = g_D . \quad (19)$$

Eq. (16) together with the energy conservation equation

$$\dot{\rho} + 3H(\rho + p) = 0 , \quad (20)$$

govern the evolution of the Universe. Using these two equations, we find the current cosmic acceleration can be interpreted while without invoking any kind of dark energy. To show this point, we may investigate the cosmic evolution sourced by dark matter and baryon matter. Observations reveal that the Universe is highly flat in space. So in the following we shall put $K = 0$. Define

$$u \equiv \ln a . \quad (21)$$

Combining Eq. (16) and Eq. (20) we obtain the Friedmann equation:

$$8\pi\rho = \int 3 \left[-2\mathfrak{D}(x) + \frac{3x}{e^x - 1} \right] dH^2 . \quad (22)$$

Let

$$h \equiv \frac{H}{H_0} , \quad \Omega_{m0} \equiv \frac{\rho_{m0}}{\rho_0} , \quad H_D = \zeta H_0 , \quad (23)$$

then we have

$$\Omega_{m0} e^{-3u} = \int_{\infty}^x \left[4\mathfrak{D}(x) - \frac{6x}{e^x - 1} \right] \cdot \frac{\zeta^2}{x^3} dx , \quad (24)$$

and

$$x = \frac{\zeta}{h} . \quad (25)$$

Here H_0 , ρ_0 , ρ_{m0} are the Hubble parameter, total energy density and matter energy density ratio for the present-day Universe. ζ is a dimensionless free parameter. The standard cosmological model, e.g. as in Komatsu et al. [18], predicts a present matter density ratio $\Omega_{m0} = 0.25$. Using this result and taking $\zeta = 10^{-5}$, we plot the dimensionless Hubble parameter h via redshifts z in Fig. 1 for our model and the standard Λ CDM model. We find they have nearly the same behavior.

In order to show our model can lead to the cosmic acceleration, we plot the evolution of deceleration parameter q :

$$q \equiv - \left(1 + \frac{d \ln h}{du} \right) , \quad (26)$$

in Fig. 2. We find the model predicts nearly the same transition redshift $z_T = 0.8$ as the standard Λ CDM model.

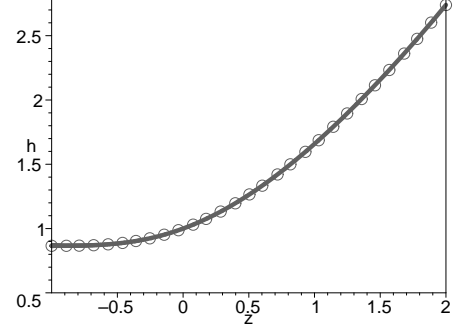


FIG. 1: The dimensionless Hubble parameter h with redshift z . The solid line is for the standard Λ CDM model. The circled line is for our model.

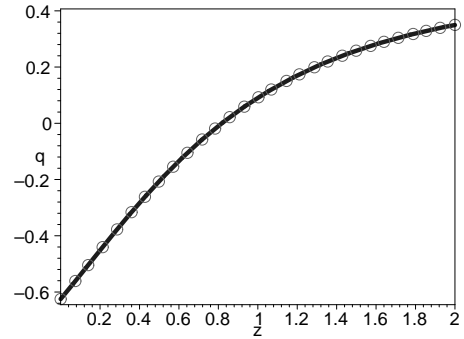


FIG. 2: The deceleration parameter q with redshift z . The solid line is for the standard Λ CDM model. The circled line is for our model.

At higher redshifts (large H), we have $x \ll 1$. Then Eq. (16) restores to the standard equation in General Relativity. So the physics at and before the radiation dominated epoch is not modified. Finally, let's calculate the Debye acceleration g_D :

$$g_D = \zeta H_0 = 10^{-15} \text{N} \cdot \text{kg}^{-1} . \quad (27)$$

It is the order of 10^{-11} of the gravitational field strength in the solar system and the order of 10^{-5} for the Galaxy system. So Eq. (10) can be perfectly written as the Newtonian law of gravity in the solar system and the Galaxy scales. To show this point in great detail, we may observe the modified Poisson's equation which is given by:

$$\nabla \cdot [\mathfrak{D}(x) \nabla \Phi] = 4\pi\rho , \quad (28)$$

with

$$x = \frac{g_D}{|\nabla \Phi|} . \quad (29)$$

Here ρ is the energy density of matter sources. Φ is the Newtonian gravitational potential. When $x \ll 1$, i.e. in the limit of strong gravitational field, we have $\mathfrak{D} = 1$ which is for the Newtonian gravity. So the deviation

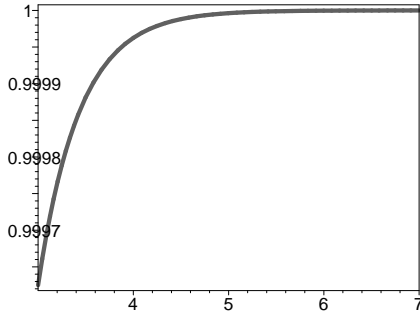


FIG. 3: The values of Debye function \mathfrak{D} with the strength of gravitational field $\log(|\nabla\phi|/g_D)$ in Galaxy.

of Eq. (28) from Newtonian gravity is embodied by the Debye function. In Fig. 3, we plot the values of Debye function in Galaxy. The strength of gravitational field in the Galaxy is around the order of 10^5 of the Debye acceleration g_D . It then follows that the Debye function $\mathfrak{D} \simeq 1$ in the Galaxy such that the Newtonian gravity emerges as a very good approximation.

III. CONCLUSION AND DISCUSSION

Statistical thermodynamics tells us the equipartition law of energy does not hold in the limit of temperature approaching zero. It is found that the Debye model is very successful in interpreting the physics of very low temperature system for most of the solid objects. Since a gravitational system can also be taken as a thermody-

namical system, we expect the equipartition law of energy for gravitational system shall break down in the limit of very low temperature (or very low gravitational field strength). Motivated by Debye model, we modified the entropic force. We find the modification can interpret the current acceleration of the Universe while without invoking any kind of dark energy.

The model is closely related the Debye acceleration g_D . We find $g_D = 10^{-15} \text{N} \cdot \text{kg}^{-1}$ is consistent with cosmic observations very well. This gravitational field strength is very much smaller than the gravitational fields, by orders of 10^{11} in solar system and by orders of 10^5 in the Galaxy. Therefore, the modified entropic force is not inconsistent with the solar system tests and the Galaxy scale observations. Then how to derive the Debye model from the specific microscopic model of spacetime? We leave this problem to future study.

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